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Some Possibilities of Determination of Yield Conditions for Cold Rolled Steel Sheets

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BIOGRAPHICAL NOTES

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KEY WORDS

Steel Sheets, Anisotropic Plasticity, Yield Conditions, Biaxial Tensile Test, Cruciform Specimen; Digital Image Correlation

ABSTRACT

Rolling of metallic materials results to anisotropy of their mechanical properties during plastic deformation. In the paper are described some possibilities of analytical and experimental determination of conditions for plastic deformation of cold rolled steel sheets. The experimental analysis is realized by biaxial tensile test with application of strain gage and optical methods. The results gained by experimental procedures are compared with yield conditions according to individual theories.

INTRODUCTION

Classical semifinished products that are used for production of thin-walled supporting systems are steel sheets. In order to increase competitiveness of steel there were recently provided massive innovations in the area of new qualities of steel sheets. Increasing of loading capacity and decreasing of structural weight is possible by using of steel sheets with smaller thickness made of materials with higher strength properties. Because the thin-walled elements are often produced by cold forming, there is an effort here, beside of reaching higher strength properties, to preserve (or increase) their plastic properties [7]. Identification of beginning of plastic deformation for plane stress state of sheets is important for modeling of cold forming processes as well as for assessment of loading capacity of thin-walled supporting structural elements. The conditions under which the material yields (beginning of plastic deformation) are called initial yield conditions. In case of material hardening during its plastic deformation we call these current yield conditions. Because the cold rolling of steel sheets changes random arrangement of crystals

to arrangement with preferred orientation, there are created textures that cause texture anisotropy of sheets during plastic deformation. Due to anisotropic plasticity of rolled sheets it is appropriate for analysis of their plastic properties to use the theory of anisotropic materials.
Yield conditions, the graphical representation of which for plane stress state are yield curves, are determined analytically or experimentally. Analytical methods for definition of yield conditions were created by von Mises in 1928, through Hill, Hosford, Gotoh, Taylor, Bishop, Barlat, Banabic etc. [1], [3], [8]. Because of approximativity of analytical treatments there are recently more spread experimental methods of determination of beginning of sheet plastic deformation under plane stress state. In such a case it is appropriate to use biaxial tensile test under which is the sheet specimen loaded independently in two perpendicular directions so that in the middle part of specimen is reached prescribed plane stress state [4], [6]. The paper is oriented to analysis of plastic properties of cold rolled sheets by analytical and experimental procedures. Biaxial tensile test is used with application of strain gage methods or by method of digital image correlation for measurement of deformations in middle part of specimen. The results of experimental treatments are compared with yield conditions for individual theories.

YIELD CONDITIONS FOR COLD ROLLED STEEL SHEETS

Anisotropic plasticity of sheets is expressed by coefficient of normal anisotropy r that represents differences of plastic properties of material in sheet plane with respect to plastic properties in direction perpendicular to sheet plane. Its value is determined by uniaxial tensile test and depends on orientation of specimen in sheet plane. Coefficient of normal anisotropy r_α represents value for a specimen oriented by angle α with respect to direction of sheet rolling. Method for determination of quantity r_α is given for example in [2].
In 1948 Hill proposed quadratic yield criterion for cold rolled sheets in the form

$$\sigma_1^2 - \frac{2r_0}{1+r_0}\sigma_1\sigma_2 + \frac{1+r_{90}}{1+r_0}\frac{r_0}{r_{90}}\sigma_2^2 = R_{e0}^2 = \frac{1+r_{90}}{1+r_0}\frac{r_0}{r_{90}}R_{e90}^2 \quad (1)$$

where r_0, r_{90}, R_{e0}, R_{e90} are normal anisotropy coefficients or yield points in direction 0°, 90°, respectively to the direction of sheet rolling.
σ_1, σ_2 - principal stresses in direction and perpendicular to direction of rolling.
The advantage of Hill (1948) yield criterion is that it is easy to understand. It explains its wide usage in practice, mainly in finite element analysis. The criterion needs only a small number of mechanical parameters to define the yield functions. For the plane stress state, three parameters are needed either the normal anisotropy coefficients r_0, r_{90} and the uniaxial yield stress R_{e90} in the perpendicular direction to the rolling direction or the normal anisotropy coefficients r_0, r_{90} mentioned above and the uniaxial yield stress R_{e0} in the rolling direction. In practice these parameters are usually obtained from uniaxial tests. Besides its advantages, the Hill (1948) criterion has also some drawbacks, for example it cannot represent anomal behaviour of some materials.
Since that time was proposed number of yield conditions for steel sheets with anisotropic plasticity [1]. In the following are given the most used ones.
In 1968 Hosford and Backofen proposed simplified form of Hill's yield condition (1) in the form

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2\frac{2r}{r+1} = R_e^2, \quad (2)$$

with average value of normal anisotropy coefficient

$$r = \frac{1}{4}\left(r_0 + r_{90} + 2r_{45}\right). \quad (3)$$

In next time were proposed non-quadratic yield conditions. In 1977 proposed Gotoh biquadratic yield condition

$$A_1\sigma_x^4 + A_2\sigma_x^3\sigma_y + A_3\sigma_x^2\sigma_y^2 + A_4\sigma_x\sigma_y^{\,3} + A_5\sigma_y^4 + \\ + \left(A_6\sigma_x^2 + A_7\sigma_x\sigma_y + A_8\sigma_y^2\right)\tau_{xy}^2 + A_9\tau_{xy}^4 = R_{e0}^4 . \quad (4)$$

If we take into account that A_1=1 then coefficients A_2 up to A_5 are determined using the equations

$$A_2 = -\frac{4r_0}{1+r_0},\ A_5 = \left(\frac{R_{e0}}{R_{e90}}\right)^4,\ A_4 = -\frac{4A_5 r_{90}}{1+r_{90}}, \\ A_3 = \left(\sigma_{bi}/R_{e0}\right)^{-4} - \left(A_1 + A_2 + A_4 + A_5\right), \quad (5)$$

where σ_{bi} is an equi-biaxial yield stress under plane stress.

To determine A_6 up to A_9, there is necessary to know r - value and the yield stress at 45° and at 22,5° or 67,5° to the rolling direction of sheet metal.

Hosford's yield condition was published in 1979 and it has a form

$$r_{90}\sigma_x^{\alpha} + r_0\sigma_y^{\alpha} + r_{90}r_0\left(\sigma_x - \sigma_y\right)^{\alpha} = r_{90}\left(1 + r_0\right)R_{e0}^{\alpha} \qquad (6)$$

where exponent a is determined on the base of crystalline texture.

Barlat and Lian in 1989 included into yield condition also shear stresses and they had relation

$$b\left(k_1 + k_2\right)^m + b\left(k_1 - k_2\right)^m + \left(2 - b\right)\left(2k_2\right)^m = 2R_e^m \ ,(7)$$

where

$$k_1 = \frac{\sigma_x + h\sigma_y}{2}, \qquad k_2 = \sqrt{\left(\frac{\sigma_x - h\sigma_y}{2}\right)^2 + p^2\tau_{xy}^2} \ . \qquad (8)$$

In the equations (7) and (8) are b, h, p and m the material parameters.

Barlat in 1991 proposed yield criterion

$$f = \left|S_1 - S_2\right|^{\alpha} + \left|S_2 - S_3\right|^{\alpha} + \left|S_3 - S_1\right|^{\alpha} = 2R_e^{\alpha}, \qquad (9)$$

where S_1, S_2 and S_3 are eigenvalues of matrix S defined with respect to Cauchy stress tensor components

$$\mathbf{S} = \begin{bmatrix} \frac{c_3(\sigma_x+\sigma_y)-c_2(\sigma_z+\sigma_x)}{3} & c_6\tau_{xy} & c_5\tau_{zx} \\ c_6\tau_{xy} & \frac{c_1(\sigma_y+\sigma_z)-c_3(\sigma_x+\sigma_y)}{3} & c_4\tau_{zy} \\ c_5\tau_{zx} & c_4\tau_{zy} & \frac{c_2(\sigma_z+\sigma_x)-c_1(\sigma_y+\sigma_z)}{3} \end{bmatrix} \qquad (10)$$

Parameters c_1 to c_6 are anisotropy coefficients that can be determined from uniaxial tensile tests in directions 0°, 45° and 90°, respectively, with respect to rolling direction. Exponent α depends on anisotropy of material.

Hill proceeds in searching of new yield conditions and the most important is a theory published in 1990 and in 1993

$$\begin{aligned}&(\sigma_x+\sigma_y)^m + (\sigma_{bi}^m/\tau^m)\left\{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2\right\}^{m/2} + \\ &+(\sigma_x^2+\sigma_y^2+2\tau_{xy}^2)^{(m/2)-1}\left\{-2a(\sigma_x^2-\sigma_y^2) + b(\sigma_x-\sigma_y)^2\right\} = (2\sigma_{bi})^m ,\end{aligned} \qquad (11)$$

$$\frac{\sigma_1^2}{R_{e0}^2} - \frac{c\sigma_1\sigma_2}{R_{e0}R_{e90}} + \frac{\sigma_2^2}{R_{e90}^2} + \left[(p+q) - \frac{p\sigma_1 + q\sigma_2}{\sigma_{bi}}\right]\frac{\sigma_1\sigma_2}{R_{e0}R_{e90}} = 1, \qquad (12)$$

where a, b, p, q, c and m are non-dimensional material parameters. Quantity τ depends on quantities σ_{bi} and R_{e45}.

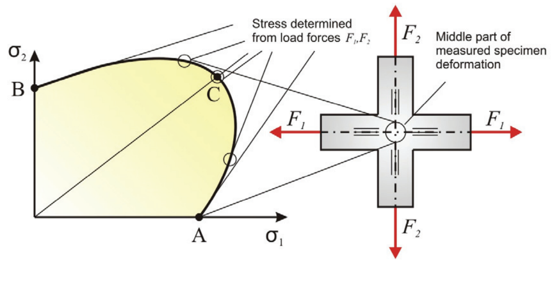
Procedure for determination of coefficients in yield conditions described by equations (4), (6), (7), (9), (10), (11) and (12) is given in [1], as well as in original publications of authors. Graphical representation of yield conditions in stress coordinate system are yield surfaces or yield curves.

ANALYSIS OF DEFORMATIONS AND STRESSES IN STEEL SHEETS BY EXPERIMENTAL METHODS

For deformation and stress analysis of machine parts and equipments are frequently used methods of experimental mechanics [11]. Experimental determination of yield curves with coordinates $\sigma_1 \geq 0$, $\sigma_2 \geq 0$ (Fig. 1a) can be realized by biaxial tensile test (Fig. 1b).

Biaxial tensile test was developed with the aim to model stress states in whole first quadrant ($\sigma_1 \geq 0$, $\sigma_2 \geq 0$). Cruciform specimen is loaded by tension in two perpendicular directions (Fig. 1b) and the plastic deformation is measured in middle part of cruciform specimen. In various states of plane stress (different ratios of tension stresses) which are ensured by change of load forces can be determined points of yield curve in the whole first quadrant. Advantage of the test is that it is universal for all sheet types and it is possible to determine strengthening of material. The geometry of specimen, stress and strain distribution in location of deformation measurement as well as method of stress computation in specimen from the loading forces has great impact on results of biaxial tensile test and their representation. Because at present do not exist any norm (or directive) for determination of the shape and dimensions of cruciform specimens, most laboratories create their own conditions. Geometry of specimen has to be chosen with the aim to reach maximal area of homogeneous stress in middle part of specimen, minimal influence of shear stress invoked by load, uniform stress and strain distribution in middle part of specimen as well as minimal stress differences in middle part in comparison to nominal stresses gained by dividing of load forces by cross-section area in the most narrow part of specimen's arm.

It is possible to use finite element method for shape optimization of cruciform specimens. Result of shape optimization of cruciform specimen is shown on Fig. 2. Distribution of stresses was determined in the middle area of specimen with diameter 30 mm for force ratios $F_1 : F_2 = 1$, $F_1 : F_2 = 2$ and $F_2 : F_1 = 2$ in arms of specimen for plastic strains ε_S^p =0,002; 0,005; 0,01; 0,02 and 0,03 in the middle of specimen. In Fig. 3 are illustrated typical stress distribution σ_1, σ_2 for plastic strain ε_S^p=0,01 in middle of specimen according to Fig. 2, for forces ratio $F_1 : F_2 = 2$ and using of Barlat yield criterion (9) developed in 1991.



a) points on the yield curve b) cruciform specimen

Fig. 1 *Experimental determination of yield curves by biaxial tensile test*

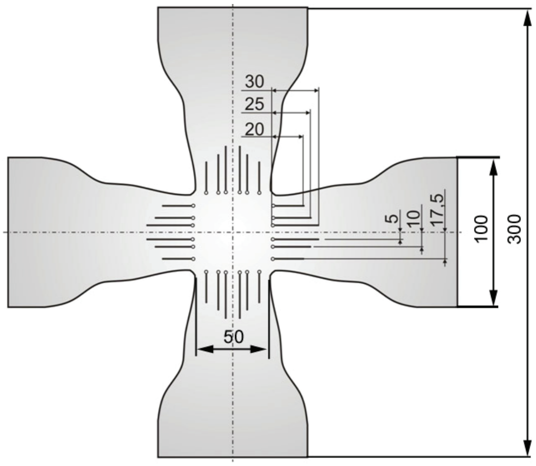


Fig. 2 *Optimized shape of cruciform specimen*

For experimental determination of yield curves by biaxial tensile test was proposed full experimental chain that consists of hydraulic equipment for biaxial loading of cruciform specimen with a possibility to measure loading forces and also a system for scanning deformation in middle part of specimen during its biaxial tension. Hydraulic loading equipment (Fig.4) consists of four hydraulic cylinders placed in pairs against each other. On the connecting rods of hydraulic cylinders are fixing jaws placed in slides [2].

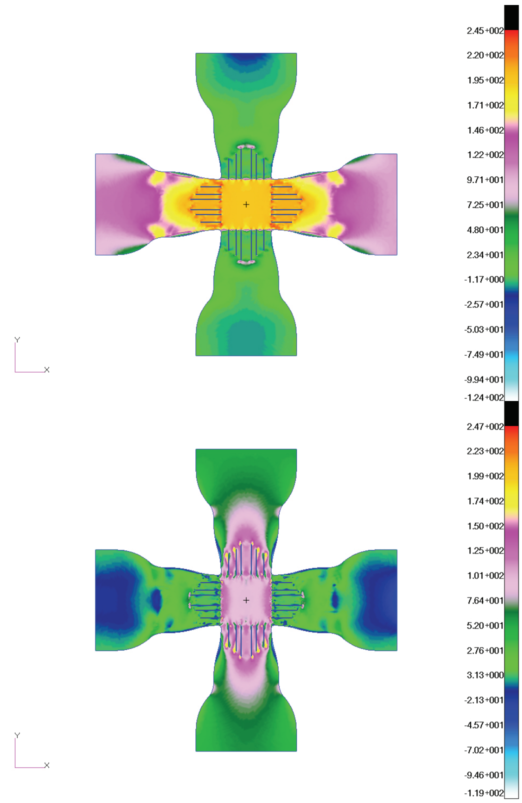


Fig. 3 *Distribution of stresses* σ_1, σ_2 *in specimen according to Fig. 2 with considering Barlat plastic condition*

In order to ensure the uniformity of displacements and accordingly constant velocity of connecting rods is into loading equipment included pantograph that ensures the same displacement of cylinders along axis and fixating of specimen's middle part. In connecting rods of hydraulic cylinders are placed strain gage dynamometers for measuring of forces F_1, F_2 in directions of cruciform specimen axis. For the measurement of deformations in middle part of cruciform specimen can be used

a) Electrical resistance strain gages applied directly in middle part of cruciform specimen

For the measurement is very appropriate to use strain gage rosette for high strains (e.g. type EP-08-062TT-120-Vishay). Gages are suitable for the measurement of deformations in ranges ±10% for temperature from -269° to 230°C [10].

b) Contact biaxial extensometer

For the application of contact measurement method of strain in the middle part of cruciform specimen was suggested and designed contact biaxial extensometer (Fig. 5). Extensometer consists of two perpendicular elastic members with a points. Elastic members are pushed down into the sheet's surface by points so that during deformation there is a relative shift of points. It causes deformation of elastic members with strain gages. On every elastic member are applied four strain gages connected into Wheatston bridge with exclusion of tension and temperature. Measured length of extensometer $\ell = 30$ mm is determined by distances of points and was proposed such a way that for a given type of cruciform specimen extensometer reads deformations in the area of their homogeneous distribution. Detailed description of contact extensometer is in literature [2].

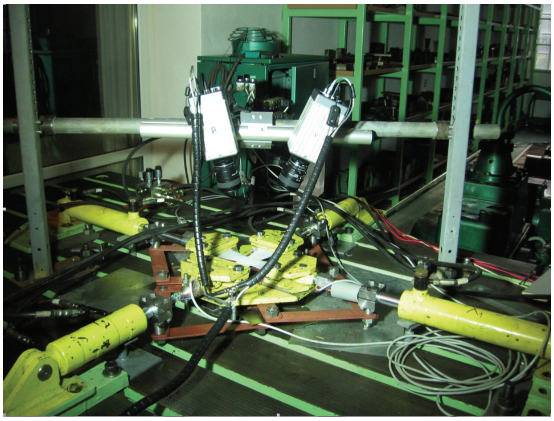


Fig. 4 *Hydraulic load equipment with system Q-450*

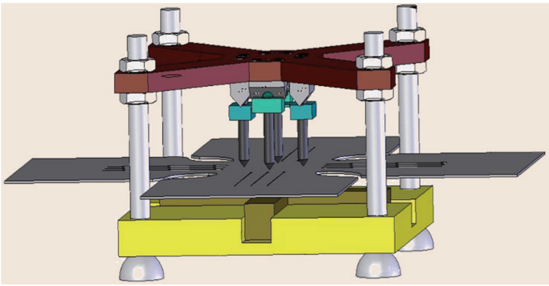


Fig. 5 *Position of biaxial extensometer during measurement*

c) Method of digital image correlation

Method of digital image correlation (DIC) is a modern optical method used frequently for analysis strains on the surface of objects [5], [9]. In the case of classical image correlation the deformations of object are determined by CCD camera. In the process of digital image correlation are determined displacements, rotations and bending of small elements, so-called facets that are determined in reference image. Correlation algorithms can determine maximal displacement of point with precision to 1/100 pixel. Such a treatment allows determining deformation of object in a plane parallel to a plane of camera's image. For deformation analysis in 3D are used two cameras. If the object is observed from two different directions, the position of every point is focused to individual pixel in the plane of camera in question. If are known the relative positions of both cameras, magnification factor of objective and all parameters of image, we are able to compute absolute three-dimensional coordinates of every point on the object's surface in space (Fig. 6).

From the known coordinates of every point on the surface we can determine position of surface in 3D in all areas observed by two cameras. The structure of the surface has to have satisfactorily good quality in order to be able to use correlation algorithms for identical points from both cameras. It is the reason why the surface is sprayed with black and white grain structure (Fig. 7). Measurement was accomplished by optical system Q-450 from DANTEC Dynamics (Fig. 4). It is a system that allows measurement of three-dimensional displacements and deformations almost all material types and parts [10].

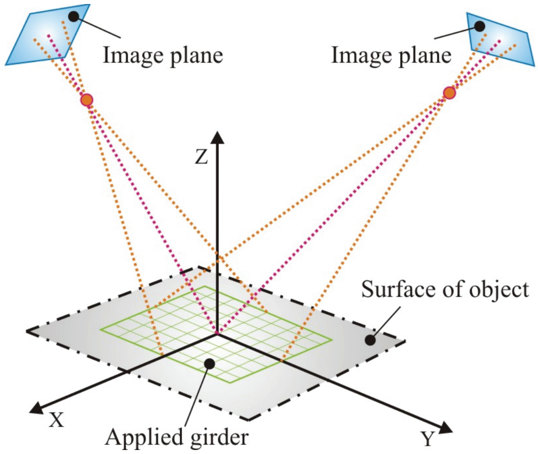


Fig. 6 *Principle of 3D pattern correlation with two cameras*

For experimental evaluation of plastic properties were used cold rolled steel sheets. In Tab. 1 are given mechanical properties of cold rolled hot dip galvanized sheet HX220PD with higher strength properties from IF steel alloyed by phosphorus. Relative direction was direction of rolling. With respect to observed range of plastic deformations was considered coefficient of normal anisotropy $r_{(5)}$ evaluated for 5% plastic deformation.

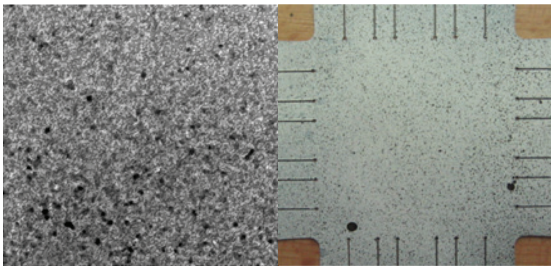


Fig. 7 *Black and white grain structure on object's surface*

From the measured stress-strain dependences were for given material determined points of yield curves for chosen magnitudes of plastic strains. For determination of points on yield curves was used criterion of maximal magnitude of plastic strain with plastic strains ε_k^p = 0,002; 0,005; 0,01; 0,015; 0,02 and 0,03. The experimentally determined points of yield curves were consecutively compared with analytical yield conditions.

In Fig. 8 are depicted yield curves for plastic strain ε_k^p = 0,002 for above mentioned yield conditions. They are divided into two categories. In figures are at the same time given experimentally gained values of stresses for plastic deformation ε_k^p = 0,002 that were gained by resistance strain gages positioned in the middle of specimen and by contact biaxial extensometer with biaxial tensile test for force ratios 0,5; 1 and 2 in arms of specimen.

In Fig. 9 are shown experimentally determined points of yield curves for force ratios 0,5; 1 and 2 in specimen arms, gained by method of digital image correlation for plastic strains ε_S^p = 0,002; 0,005; 0,01; 0,015; 0,02 and 0,03 in the middle of specimen compared with analytical yield conditions according to Hill (1948), Hill (1993), Gotoh (1977) and Barlat (1989) theory. In Fig. 10 is shown typical distribution and directions of principal strains in middle area of specimen for ε_S^p = 0,01 and 0,02 determined by the method of digital image correlation for force ratios in specimen's arm equal 1:1.

Material	Thickness [mm]	Direction	$R_{p0,2}$ [MPa]	R_m [MPa]	A_{80} [%]	n	$r_{(5)}$
HX220PD	0,70	0°	220	380	39,3	0,218	1,14
		45°	228	364	37,8	0,220	1,80
		90°	241	381	37,8	0,211	1,82

Tab. 1 *Mechanical properties of tested sheets*

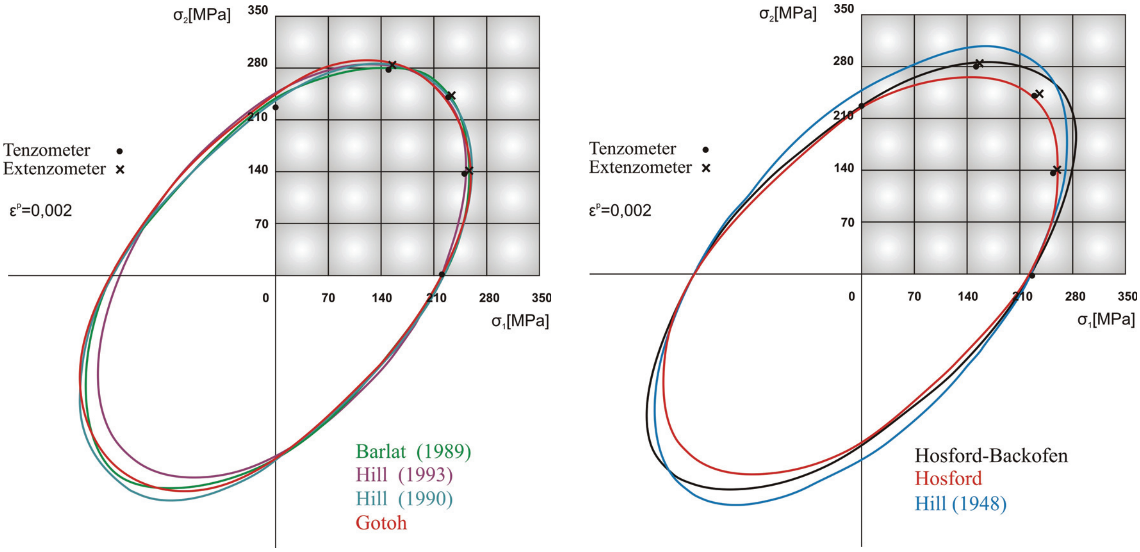


Fig. 8 *Theoretical yield curves for steel sheet of quality HX220PD and experimental values*

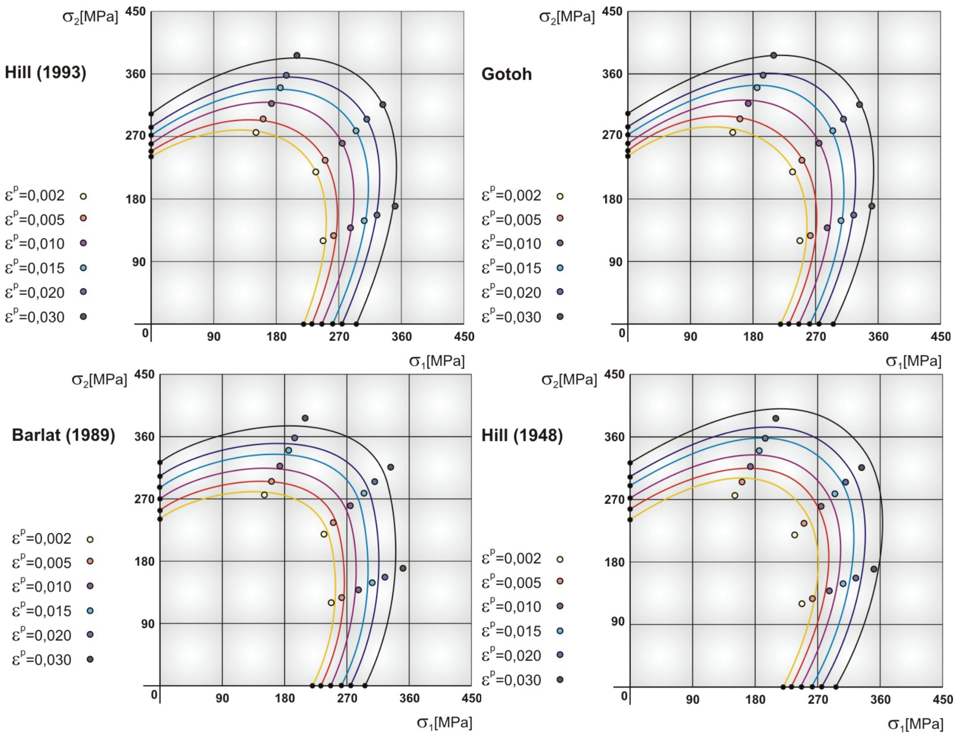


Fig. 9 *Experimental by determined points of yield curves for the steel sheets made of material HX220PD*

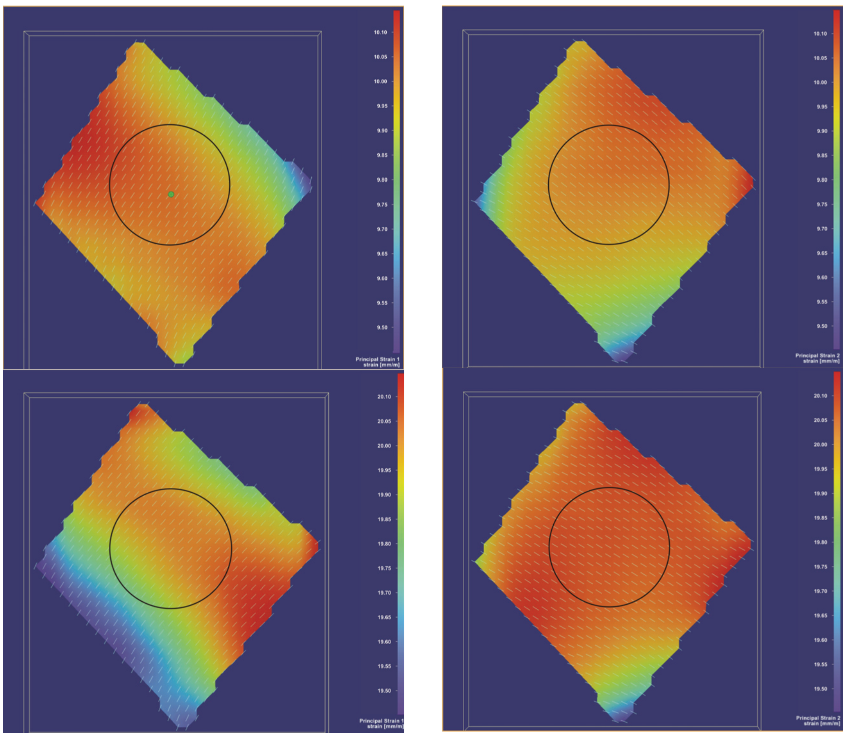


Fig. 10 *Distributions and directions of principle strains for force ratio 1:1 a) for $\varepsilon_S^p = 0{,}01$, b) for $\varepsilon_S^p = 0{,}02$*

CONCLUSIONS

For the analysis of strains and stresses in sheets with anisotropic plasticity can be used analytical, numerical or experimental methods of mechanics. In the paper are given the most frequently used yield conditions for cold rolled sheets. For experimental determination of stresses and strains is applied biaxial tensile test in order to reach plane stress state. Quantification of strains in middle part of cruciform specimen was realized by the strain gage method and method of digital image correlation. For the analysis was used cold rolled hot dip galvanized steel sheet made of IF steel with higher strength alloyed by phosphorus.

As results from Fig. 8 the best agreement of experimental values for plastic deformation 0,002 is with Hill (1993), Hill (1990), Barlat (1989) and Gotoh theory. Comparison of analytical and experimental methods showed that for plastic deformation from 0,002 to 0,03 (Fig. 9) the best agreement was reached for Hill's theory from 1993. Also Gotoh's yield condition is in a good agreement with the experiments, but for the force ratio 1:2 and 2:1 for plastic deformation under 0,01 slightly overevaluates stresses. Hill's theory from 1948, which is the most used in numerical modeling of plastic deformations, significantly overevaluate stresses mainly closely after beginning of plastic deformation. Difference in comparison to the experimental results is to 10%. Barlat theory represents good agreement with experimental results for plastic strains 0,002 and 0,005. With increased value of plastic strain is the difference increased and for value 0,03 reaches difference approximately 5%.

In Fig.10 are the fields of plastic deformations gained with the optical system Q–450 that uses digital image correlation. In the middle part is roughly homogeneous deformation that supports correctness of cruciform specimen design. On the base of first opinions gained from using of this measurement method can be stated that it is suitable for determination of plastic deformations in plane stress state.

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